NEAPIBRĖŽTINIAI INTEGRALAI

INTEGRAVIMO METODAI I

<u>Integravimas keičiant kintamąjį</u>

1. $\int \frac{\cos\varphi}{a^2+\sin^2\varphi}\cdot d\varphi = \int \frac{d(\sin\varphi)}{a^2+\sin^2\varphi} = [\sin\varphi = t] = \int \frac{dt}{a^2+t^2} = \frac{1}{a}\cdot arctg\frac{t}{a} + C = \frac{1}{a}\cdot arctg\frac{\sin\varphi}{a} + C.$

2.

$$\int \frac{\sqrt{x^2-a^2}}{x}\cdot dx = \begin{bmatrix} x^2-a^2=t^2 \\ x=\sqrt{t^2+a^2} \\ dx=\frac{t\cdot dt}{\sqrt{t^2+a^2}} \end{bmatrix} = \int \frac{t}{\sqrt{t^2+a^2}}\cdot\frac{t\cdot dt}{\sqrt{t^2+a^2}} = \int \frac{t^2}{t^2+a^2}\cdot dt = \int \frac{t^2+a^2-a^2}{t^2+a^2}\cdot dt =$$

$$= \int \frac{t^2+a^2}{t^2+a^2}\cdot dt - \int \frac{a^2}{t^2+a^2}\cdot dt = \int dt - a^2\cdot\int \frac{dt}{t^2+a^2} = t - a^2\cdot\frac{1}{a}\cdot arctg\frac{t}{a} + C = \sqrt{x^2-a^2} - a\cdot arctg\frac{\sqrt{x^2-a^2}}{a} + C$$

3. $\int \frac{1}{(1+x^2)\cdot arctg^2 x}\cdot dx = \left[arctgx = t,\ dt = \frac{dx}{1+x^2}\right] = \int \frac{dt}{t^2} = -\frac{1}{t} + C = -\frac{1}{arctgx} + C.$

4.

$$\int (1-\sin 2x)^2\cdot\cos 2x\cdot dx = [\sin 2x = t,\ dt = 2\cdot\cos 2x\cdot dx] = \int (1-t)^2\cdot\frac{1}{2}\cdot dt = -\frac{1}{2}\cdot\frac{1}{2+1}\cdot(1-t)^{2+1} + C =$$

$$= -\frac{1}{6}\cdot(1-t)^3 + C = -\frac{1}{6}\cdot(1-\sin 2x)^3 + C.$$

5. $\int \frac{dx}{x\cdot(4+\ln^2 x)} = \left[\ln x = t,\ dt = \frac{dx}{x}\right] = \int \frac{dt}{2^2+t^2} = \frac{1}{2}\cdot arctg\frac{t}{2} + C = \frac{1}{2}\cdot arctg\frac{\ln x}{2} + C.$

6. $\int e^{3x+1}\cdot d(3x+1) = [3x+1 = t] = \int e^t\cdot dt = e^t + C = e^{3x+1} + C.$

7. $\int \cos(x-2)\cdot d(x-2) = [x-2 = t] = \int \cos t\cdot dt = \sin t + C = \sin(x-2) + C.$

8. $\int tgx\cdot dx = \int \frac{\sin x}{\cos x}\cdot dx = -\int \frac{d(\cos x)}{\cos x} = [\cos x = t] = -\int \frac{dt}{t} = -\ln|t| + C = -\ln|\cos x| + C.$

Integravimas dalimis

$$\int u\cdot dv = u\cdot v - \int v\cdot du$$

9.

$$\int \ln\left(x+\sqrt{1+x^2}\right)\cdot dx = \ln\left(x+\sqrt{1+x^2}\right)\cdot x - \int x\cdot d\left(\ln\left(x+\sqrt{1+x^2}\right)\right) =$$

$$= \ln\left(x+\sqrt{1+x^2}\right)\cdot x - \int x\cdot \frac{1}{x+\sqrt{1+x^2}}\cdot\left(1+\frac{1}{2\sqrt{1+x^2}}\cdot 2x\right)\cdot dx =$$

$$= \ln\left(x+\sqrt{1+x^2}\right)\cdot x - \int x\cdot \frac{1}{x+\sqrt{1+x^2}}\cdot\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\cdot dx = \ln\left(x+\sqrt{1+x^2}\right)\cdot x - \int \frac{x}{\sqrt{1+x^2}}\cdot dx =$$

$$= \ln\left(x+\sqrt{1+x^2}\right)\cdot x - \int \frac{1}{\sqrt{1+x^2}}\cdot d\left(\frac{1}{2}x^2\right) = \ln\left(x+\sqrt{1+x^2}\right)\cdot x - \frac{1}{2}\cdot\int\frac{d\left(x^2+1\right)}{\sqrt{1+x^2}} =$$

$$= \ln\left(x+\sqrt{1+x^2}\right)\cdot x - \frac{1}{2}\cdot 2\sqrt{1+x^2} + C = \ln\left(x+\sqrt{1+x^2}\right)\cdot x - \sqrt{1+x^2} + C.$$

10.

$$\int x\cdot\sin 3x\cdot dx = \frac{1}{3}\int x\cdot\sin(3x)\cdot d(3x) = -\frac{1}{3}\int x\cdot d(\cos 3x) = -\frac{1}{3}x\cdot\cos 3x + \frac{1}{3}\int\cos 3x\cdot dx =$$

$$= -\frac{1}{3}x\cdot\cos 3x + \frac{1}{9}\int\cos 3x\cdot d(3x) = -\frac{1}{3}x\cdot\cos 3x + \frac{1}{9}\sin 3x + C.$$

11.

$$\int\frac{\ln x}{x^2}\cdot dx = \int\ln x\cdot d\left(-\frac{1}{x}\right) = -\frac{1}{x}\cdot\ln x + \int\frac{1}{x}\cdot d(\ln x) = -\frac{1}{x}\cdot\ln x + \int\frac{1}{x}\cdot\frac{1}{x}dx = -\frac{1}{x}\cdot\ln x + \int\frac{1}{x^2}\cdot dx =$$

$$= -\frac{1}{x}\cdot\ln x - \frac{1}{x} + C = -\frac{1}{x}\cdot(\ln x + 1) + C.$$

12.

$$\int x\cdot e^{-2x}\cdot dx = \int x\cdot d\left(-\frac{1}{2}e^{-2x}\right) = -\frac{1}{2}x\cdot e^{-2x} + \frac{1}{2}\int e^{-2x}\cdot dx = -\frac{1}{2}x\cdot e^{-2x} + \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\cdot e^{-2x} + C =$$

$$= -\frac{1}{4}e^{-2x}\cdot(2x+1) + C.$$

13.

$$\int x^3\cdot\ln x\cdot dx = \frac{1}{4}\int\ln x\cdot d\left(x^4\right) = \frac{1}{4}x^4\cdot\ln x - \frac{1}{4}\int x^4\cdot d(\ln x) = \frac{1}{4}x^4\cdot\ln x - \frac{1}{4}\int x^4\cdot\frac{1}{x}\cdot dx =$$

$$= \frac{1}{4}x^4\cdot\ln x - \frac{1}{4}\int x^3\cdot dx = \frac{1}{4}x^4\cdot\ln x - \frac{1}{4}\cdot\frac{1}{4}\cdot x^4 + C = \frac{1}{4}x^4\cdot\left(\ln x - \frac{1}{4}\right) + C.$$

14.

$$\int arctg(x)\cdot dx = x\cdot arctg(x) - \int x\cdot d(arctg(x)) = x\cdot arctg(x) - \int x\cdot\frac{1}{1+x^2}\cdot dx = x\cdot arctg(x) - \int\frac{d\left(\frac{1}{2}x^2\right)}{1+x^2} =$$

$$= x\cdot arctg(x) - \frac{1}{2}\cdot\int\frac{d(1+x^2)}{1+x^2} = x\cdot arctg(x) - \frac{1}{2}\cdot\ln\left|1+x^2\right| + C.$$

15.

$$\int e^{-x}\cdot\cos x\cdot dx = \int e^{-x}\cdot d(\sin x) = e^{-x}\cdot\sin x - \int\sin x\cdot d\left(e^{-x}\right) = e^{-x}\cdot\sin x - \int\sin x\cdot\left(-e^{-x}\right)\cdot dx =$$

$$= e^{-x}\cdot\sin x + \int e^{-x}\cdot\sin x\cdot dx = e^{-x}\cdot\sin x + \int e^{-x}\cdot d(-\cos x) = e^{-x}\cdot\sin x - e^{-x}\cdot\cos x + \int\cos x\cdot d\left(e^{-x}\right) =$$

$$= e^{-x}\cdot\sin x - e^{-x}\cdot\cos x + \int\cos x\cdot\left(-e^{-x}\right)\cdot dx = e^{-x}\cdot\sin x - e^{-x}\cdot\cos x - \int e^{-x}\cdot\cos x\cdot dx$$

Lygybės kairėje ir dešinėje pusėje gavome pradinį integralą:

$$\int e^{-x}\cdot\cos x\cdot dx = e^{-x}\cdot\sin x - e^{-x}\cdot\cos x - \int e^{-x}\cdot\cos x\cdot dx.$$

Išsprendę pastarąją lygtį, gauname:

$$\int e^{-x}\cdot\cos x\cdot dx = \frac{1}{2}\cdot\left(e^{-x}\cdot\sin x - e^{-x}\cdot\cos x\right) + C.$$

16.

$$\int\cos\ln x\cdot dx = \cos\ln x\cdot x - \int x\cdot d(\cos\ln x) = \cos\ln x\cdot x - \int x\cdot(-\sin\ln x)\cdot\frac{1}{x}\cdot dx =$$

$$= \cos\ln x\cdot x + \int\sin\ln x\cdot dx = \cos\ln x\cdot x + \sin\ln x\cdot x - \int x\cdot d(\sin\ln x) =$$

$$= \cos\ln x\cdot x + \sin\ln x\cdot x - \int x\cdot\cos\ln x\cdot\frac{1}{x}\cdot dx = \cos\ln x\cdot x + \sin\ln x\cdot x - \int\cos\ln x\cdot dx$$

Lygybės kairėje ir dešinėje pusėje gavome pradinį integralą:

$$\int\cos\ln x\cdot dx = \cos\ln x\cdot x + \sin\ln x\cdot x - \int\cos\ln x\cdot dx.$$

Išsprendę pastarąją lygtį, gauname:

$$\int\cos\ln x\cdot dx = \frac{1}{2}\cdot(\cos\ln x\cdot x + \sin\ln x\cdot x) + C.$$

17.

$$\int \frac{2x+1}{\sin^2 x}\cdot dx = \int (2x+1)\cdot d(-ctg\ x) = -ctg\ x\cdot(2x+1) + \int ctg\ x\cdot d(2x+1) = -ctg\ x\cdot(2x+1) + 2\int ctg\ x\cdot dx =$$

$$= -ctg\ x\cdot(2x+1) + 2\int \frac{\cos x}{\sin x}\cdot dx = -ctg\ x\cdot(2x+1) + 2\int \frac{d(\sin x)}{\sin x} = -ctg\ x\cdot(2x+1) + 2\cdot\ln|\sin x| + C.$$

18.

$$\int x\cdot\ln^2 x\cdot dx = \frac{1}{2}\int \ln^2 x\cdot d(x^2) = \frac{1}{2}x^2\cdot\ln^2 x - \frac{1}{2}\int x^2\cdot d(\ln^2 x) = \frac{1}{2}x^2\cdot\ln^2 x - \frac{1}{2}\int x^2\cdot 2\ln x\cdot\frac{1}{x}\cdot dx =$$

$$= \frac{1}{2}x^2\cdot\ln^2 x - \int x\cdot\ln x\cdot dx = \frac{1}{2}x^2\cdot\ln^2 x - \frac{1}{2}\int \ln x\cdot d(x^2) = \frac{1}{2}x^2\cdot\ln^2 x - \frac{1}{2}x^2\cdot\ln x + \frac{1}{2}\int x^2\cdot d(\ln x) =$$

$$= \frac{1}{2}x^2\cdot\ln^2 x - \frac{1}{2}x^2\cdot\ln x + \frac{1}{2}\int x^2\cdot\frac{1}{x}\cdot dx = \frac{1}{2}x^2\cdot\ln^2 x - \frac{1}{2}x^2\cdot\ln x + \frac{1}{2}\int x\cdot dx = \frac{1}{2}x^2\cdot\ln^2 x - \frac{1}{2}x^2\cdot\ln x + \frac{1}{4}x^2 + C.$$

19.

$$\int e^x\cdot x^2\cdot dx = \int x^2\cdot d(e^x) = x^2\cdot e^x - \int e^x\cdot d(x^2) = x^2\cdot e^x - \int e^x\cdot 2x\cdot dx = x^2\cdot e^x - 2\cdot\int x\cdot d(e^x) =$$

$$= x^2\cdot e^x - 2\cdot x\cdot e^x + 2\cdot\int e^x\cdot dx = x^2\cdot e^x - 2x\cdot e^x + 2\cdot e^x + C.$$

20.

$$\int \frac{\arcsin x}{x^2}\cdot dx = \int \arcsin x\cdot d\left(-\frac{1}{x}\right) = -\arcsin x\cdot\frac{1}{x} + \int \frac{1}{x}\cdot d(\arcsin x) = -\arcsin x\cdot\frac{1}{x} + \int \frac{1}{x}\cdot\frac{1}{\sqrt{1-x^2}}\cdot dx =$$

$$= -\arcsin x\cdot\frac{1}{x} + \int \frac{dx}{x^2\cdot\sqrt{\frac{1}{x^2}-1}} = -\arcsin x\cdot\frac{1}{x} - \int \frac{d\left(\frac{1}{x}\right)}{\sqrt{\frac{1}{x^2}-1}} = -\arcsin x\cdot\frac{1}{x} - \ln\left|\frac{1}{x} + \sqrt{\frac{1}{x^2}-1}\right| + C.$$

Trigonometrinių funkcijų integravimas

21. $\int \frac{\sin x - \cos x}{\sin x + \cos x} \cdot dx = \int \frac{d(-\cos x - \sin x)}{\sin x + \cos x} = -\int \frac{d(\sin x + \cos x)}{\sin x + \cos x} = -\ln|\sin x + \cos x| + C.$

22.

$$\int \frac{\sin 2x \cdot dx}{\sin^2 x + 7\sin x + 13} = \int \frac{2 \cdot \sin x \cdot \cos x \cdot dx}{\sin^2 x + 7\sin x + 13} = \int \frac{2 \cdot \sin x \cdot d(\sin x)}{\sin^2 x + 7\sin x + 13} = [\sin x = t] = \int \frac{2t \cdot dt}{t^2 + 7t + 13} =$$

$$= \int \frac{2t + 7 - 7}{t^2 + 7t + 13} \cdot dt = \int \frac{2t + 7}{t^2 + 7t + 13} \cdot dt + \int \frac{-7}{t^2 + 7t + 13} \cdot dt =$$

$$= \int \frac{d(t^2 + 7t + 13)}{t^2 + 7t + 13} - 7 \cdot \int \frac{dt}{t^2 + 2 \cdot \frac{7}{2} t + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 13} = \ln|t^2 + 7t + 13| - 7 \cdot \int \frac{d\left(t + \frac{7}{2}\right)}{\left(t + \frac{7}{2}\right)^2 + \frac{3}{4}} =$$

$$= \ln|t^2 + 7t + 13| - 7 \cdot \frac{1}{\sqrt{\frac{3}{4}}} \cdot arctg\left(\frac{t + \frac{7}{2}}{\sqrt{\frac{3}{4}}}\right) + C = \ln|t^2 + 7t + 13| - \frac{14}{\sqrt{3}} \cdot arctg\left(\frac{2t + 7}{\sqrt{3}}\right) + C =$$

$$= \ln|\sin^2 x + 7\sin x + 13| - \frac{14}{\sqrt{3}} \cdot arctg\left(\frac{2\sin x + 7}{\sqrt{3}}\right) + C$$

23.

$$\int \frac{dx}{\sin x} = \int \frac{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}}{2\sin \frac{x}{2} \cdot \cos \frac{x}{2}} \cdot dx = \int \frac{\cos^2 \frac{x}{2}}{2\sin \frac{x}{2} \cdot \cos \frac{x}{2}} \cdot dx + \int \frac{\sin^2 \frac{x}{2}}{2\sin \frac{x}{2} \cdot \cos \frac{x}{2}} \cdot dx =$$

$$= \int \frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} \cdot d\left(\frac{x}{2}\right) + \int \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \cdot d\left(\frac{x}{2}\right) = \int \frac{d\left(\sin \frac{x}{2}\right)}{\sin \frac{x}{2}} - \int \frac{d\left(-\cos \frac{x}{2}\right)}{\cos \frac{x}{2}} = \ln\left|\sin \frac{x}{2}\right| - \ln\left|\cos \frac{x}{2}\right| + C = \ln\left|\frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}\right| + C =$$

$$= \ln\left|tg \frac{x}{2}\right| + C.$$

24.

$$\int \frac{\sin^3 x \cdot dx}{\cos^4 x} = \int \frac{\sin x \cdot \sin^2 x \cdot dx}{\cos^4 x} = \int \frac{\sin x \cdot (1 - \cos^2 x) \cdot dx}{\cos^4 x} = \int \frac{\sin x \cdot dx}{\cos^4 x} + \int \frac{\sin x \cdot (-\cos^2 x) \cdot dx}{\cos^4 x} =$$

$$= -\int \frac{d(\cos x)}{\cos^4 x} + \int \frac{d(\cos x)}{\cos^2 x} = \frac{1}{3} \cdot \frac{1}{\cos^3 x} - \frac{1}{\cos x} + C$$

25.

$$\int \frac{dx}{\cos x+2\sin x+3}=\int \frac{dx}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}+4\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}+3\cos^2\frac{x}{2}+3\sin^2\frac{x}{2}}=$$

$$=\int \frac{dx}{2\cdot\cos^2\frac{x}{2}\cdot\left(tg^2\frac{x}{2}+2\cdot tg\frac{x}{2}+2\right)}=\int \frac{d\left(tg\frac{x}{2}\right)}{tg^2\frac{x}{2}+2\cdot tg\frac{x}{2}+2}=\left[tg\frac{x}{2}=t\right]=\int \frac{dt}{t^2+2\cdot t+2}=\int \frac{dt}{(t+1)^2+1}=$$

$$=arctg(t+1)+C=arctg\left(tg\frac{x}{2}+1\right)+C$$

26.

$$\int \frac{1+\sin x+\cos x}{1+\sin x-\cos x}\cdot dx=\int \frac{\sin x+\cos x}{1+\sin x-\cos x}\cdot dx+\int \frac{1}{1+\sin x-\cos x}\cdot dx=$$

$$=\int \frac{d(-\cos x+\sin x)}{1+\sin x-\cos x}+\int \frac{1}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}-\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}\cdot dx=$$

$$=\int \frac{d(1+\sin x-\cos x)}{1+\sin x-\cos x}+\int \frac{1}{2\sin^2\frac{x}{2}+2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\cdot dx=\ln|1+\sin x-\cos x|+\int \frac{1}{2\sin^2\frac{x}{2}\cdot\left(1+\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)}\cdot dx=$$

$$=\ln|1+\sin x-\cos x|-\int \frac{d(ctgx)}{1+ctg\frac{x}{2}}=\ln|1+\sin x-\cos x|-\ln\left|1+ctg\frac{x}{2}\right|+C$$

27.

$$\int \frac{2tgx+3}{\sin^2 x+2\cos^2 x}\cdot dx=\int \frac{2tgx+3}{\cos^2 x\cdot\left(\frac{\sin^2 x}{\cos^2 x}+2\right)}\cdot dx=\int \frac{2tgx+3}{tg^2x+2}\cdot d(tgx)=[tgx=t]=$$

$$=\int \frac{2t+3}{t^2+2}\cdot dt=\int \frac{2t}{t^2+2}\cdot dt+\int \frac{3}{t^2+2}\cdot dt=\int \frac{d(t^2+2)}{t^2+2}+3\cdot\int \frac{dt}{t^2+2}=\ln|t^2+2|+3\cdot\frac{1}{\sqrt{2}}\cdot arctg\frac{t}{\sqrt{2}}+C=$$

$$=\ln|tg^2x+2|+\frac{3\sqrt{2}}{2}\cdot arctg\frac{\sqrt{2}tgx}{2}+C$$

28.

$$\int \cos^4 x\cdot dx=\int\left(\frac{1}{2}\cdot(1+\cos 2x)\right)^2\cdot dx=\frac{1}{4}\cdot\int(1+2\cos 2x+\cos^2 2x)\cdot dx=$$

$$=\frac{1}{4}\cdot\int dx+\frac{1}{4}\cdot\int 2\cos 2x\cdot dx+\frac{1}{4}\cdot\int\cos^2 2x\cdot dx=\frac{1}{4}\cdot x+\frac{1}{4}\cdot\sin 2x+\frac{1}{4}\cdot\int\frac{1}{2}\cdot(1+\cos 4x)\cdot dx=$$

$$=\frac{1}{4}\cdot x+\frac{1}{4}\cdot\sin 2x+\frac{1}{8}\cdot x+\frac{1}{32}\cdot\sin 4x+C$$

29.

$$\int \frac{dx}{\sin^3 x} = \int \frac{dx}{\left(2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}\right)^3} = \frac{1}{8}\cdot\int \frac{dx}{\frac{\sin^3\frac{x}{2}}{\cos^3\frac{x}{2}}\cdot\cos^6\frac{x}{2}} = \frac{1}{8}\cdot\int \frac{dx}{tg^3\frac{x}{2}\cdot\cos^4\frac{x}{2}\cdot\cos^2\frac{x}{2}} =$$

$$= \frac{1}{8}\cdot\int \frac{\left(1+tg^2\frac{x}{2}\right)^2}{tg^3\frac{x}{2}\cdot\cos^2\frac{x}{2}}\cdot dx = \frac{1}{4}\cdot\int \frac{\left(1+tg^2\frac{x}{2}\right)^2}{tg^3\frac{x}{2}}\cdot d\left(tg\frac{x}{2}\right) = \frac{1}{4}\cdot\int \frac{1+2tg^2\frac{x}{2}+tg^4\frac{x}{2}}{tg^3\frac{x}{2}}\cdot d\left(tg\frac{x}{2}\right) =$$

$$= \frac{1}{4}\cdot\int\left(tg^{-3}\frac{x}{2} + \frac{2}{tg\frac{x}{2}} + tg\frac{x}{2}\right)\cdot d\left(tg\frac{x}{2}\right) = \frac{1}{4}\cdot\left(-\frac{1}{2}\cdot tg^{-2}\frac{x}{2} + 2\cdot\ln\left|tg\frac{x}{2}\right| + \frac{1}{2}\cdot tg^2\frac{x}{2}\right) + C$$

30.

$$\int \frac{dx}{\sin^5 x\cdot\cos^3 x} = \left[\begin{array}{l} t = tgx \\ dx = \frac{dt}{1+t^2} \\ \cos x = \frac{1}{\sqrt{1+t^2}} \\ \sin x = \frac{t}{\sqrt{1+t^2}} \end{array}\right] = \int \frac{\frac{dt}{1+t^2}}{\left(\frac{t}{\sqrt{1+t^2}}\right)^5\cdot\left(\frac{1}{\sqrt{1+t^2}}\right)^3} = \int \frac{\left(1+t^2\right)^3}{t^8}\cdot dt =$$

$$= \int \frac{1+3\cdot t^2+3\cdot t^4+t^6}{t^8}\cdot dt = \int\left(t^{-8}+3\cdot t^{-6}+3\cdot t^{-4}+t^{-2}\right)\cdot dt = -\frac{1}{7}\cdot t^{-8} - \frac{3}{5}\cdot t^{-5} - t^{-3} - t^{-1} + C$$

$$= -\frac{1}{7}\cdot\frac{1}{tg^8 x} - \frac{3}{5}\cdot\frac{1}{tg^5 x} - \frac{1}{tg^3 x} - \frac{1}{tgx} + C$$